FINANCIAL VOLATILITY MEASUREMENT USING FRACTAL DIMENSION

Daniela Alexandra CRIȘAN [1]

ABSTRACT:
Financial volatility measures the variation of the price of a financial instrument over time. It is a very important instrument for both financial analysts and investors, as it is a measure of assets (un)stability. There have been previous attempts of using fractal analysis in estimating the financial volatility.
In this paper, the author will experiment the use of an image processing technique, computing the fractal "box-counting" dimension of the time series corresponding to an asset price. The fractal dimensions of 20 pairs of assets will be compared in order to describe their volatility. An original software application developed by the author for fractal processing purposes was used.

Keywords: financial volatility, fractal geometry, box-counting

1. Financial volatility

Definitions

Financial volatility measures the variation of the price of a financial instrument over time. it is a measure of its stability. It indicates how much and how quickly the value of an investment, market, or market sector changes.

Investopedia [1] explains volatility as "the amount of uncertainty or risk regarding the degree and size of changes in the value of a security". It is pointed out that there are two types of the security's evolutions:

- A higher volatility means more risk, means that a security's value potentially can be spread out over a larger range of values, leading to the possibility of dramatically price changes (in either direction), over a short period of time.
- A lower volatility means more stability, more security, means that a security's value changes at a steady pace over a period of time, they don't fluctuate dramatically.

Volatility may influence the type of investments: some investors prefer investing in non-volatile securities, such as a certificate of deposit, some others are attracted by the risk and they'll invest in more volatile securities, meaning more to short selling and other forms of hedging. A security with a volatility of 50% is considered very high risk because it has the potential to increase or decrease up to half its value.

For instance, usually, the stock price of small or newer companies tend to rise and fall more sharply over short periods of time than stock of established, stable companies. Small

[1] Associate Professor, PhD, School of Computer Science for Business Management, Romanian-American University, 1B, Expozitiei Blvd., district 1, code 012101, Bucharest, Romania. Email: crisan.daniela.alexandra@profesor.rau.ro

caps are described as more volatile, and they'll present more interest in the investors oriented toward risky investments.

In today's markets, it is also possible to trade volatility directly, through the use of derivative securities such as options and variance swaps.

Volatility does not measure the direction of price changes, nor their dispersion. Considering two financial instruments, the one with higher volatility will have larger swings in values over a given period of time, but they could have the same expected return.

Measuring the financial volatility

As shown above, it is important for an investor to determine if a particular fund has an optimal return for a certain amount of risk. Modern portfolio theory offers several ways of measuring the volatility of assets, as a measure of its exposure to risk. Some of them are statistical measuring; some others use more complex theories.

The most common way of measuring volatility is computing the standard deviation of the asset according to the disparity its returns over a period of time.

Beta indicator determines the volatility of a fund exposed to general market movements. If the beta value is close to 1, that means the fund's performance closely matches the general market. A beta greater than 1 indicates that the asset both is volatile and tends to move up and down with the market. A beta lower than 1 can indicate [2] "either an investment with lower volatility than the market, or a volatile investment whose price movements are not highly correlated with the market".

2. Fractal dimension

Fractal geometry

Fractal geometry was introduced by the French mathematician Benoit Mandelbrot, around 1970. He introduced the notion of a "fractal" in order to describe an object self-similar (invariant to some geometric transformations).

He stated that fractals (through fractal geometry) are more useful to describe natural shapes than the classic Euclidian geometry describes. In his influential book "The Fractal Geometry of Nature" [17], published in 1982, he wrote: "Clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lightning travel in a straight line".

Mandelbrot emphasized the presence of fractals anywhere in the surrounding nature, in human pursuits (music, painting, architecture), and not ultimately, in stock market prices.

He proved that price changes in financial markets did not follow a Gaussian distribution, but rather Lévy stable distributions having theoretically infinite variance.

Fractal geometry has proven to be a very valuable instrument in various fields. In image processing, its success in various fields such are: medical imaging [6][7], general image processing [8] and so. was outstanding.

Fractal dimension

A defining feature of fractal objects is that its size depends on the unit used to measure it. This phenomenon was first noticed regarding the length of the border between Spain and Portugal: the encyclopaedias of both countries published two different lengths: the Spanish encyclopaedia border length was 987 km, while the Portuguese encyclopaedia estimated at 1214 km. The explanation for the strange phenomenon consists in using two different units of measure, unit less have to go through more of the border details and, therefore, to obtain greater measures.

Several ways of computing the fractal dimension were introduced: the most common is the Hausdorff dimension that defines the fractal dimension D as a fractional number:

$$D \approx \frac{\log\left(N(s)\right)}{\log\left(1/s\right)}$$

The Hausdorff approach states that the dimension of an object is related with the number *N(s)* of spheres of size *s* needed to over to object in a *DE*-dimensional Euclidian space.

$$\mathrm{N(s)} \sim 1/\mathrm{s}^{\mathrm{DE}}$$

Based on the Hausdorff dimension, a computational method has been imagined, so the fractal dimension of forms could be easily estimated using the computer. This algorithm is known as the “box-counting algorithm”. For short, it consists in successive image covering with equal-sized squares (s=2/4/8…) and counting, at each iteration, the number N(s) of squares that cover the object’s contour. Using linear regression, the slope of the curve described by (log(N(s)), log(1/s)) pairs will be computed.

The fractal dimension describes the complexity of a form. The more complex the form is, the greater its fractal dimension will be. A fractal dimension between 1 and 2 is associated to a curve, it will be close to 1 if the curve is a straight line, but it will be closer to 2 if the curve has a very complex shape, if it is very fragmented. So, the more complex the form is, the greater fractal dimension will have.

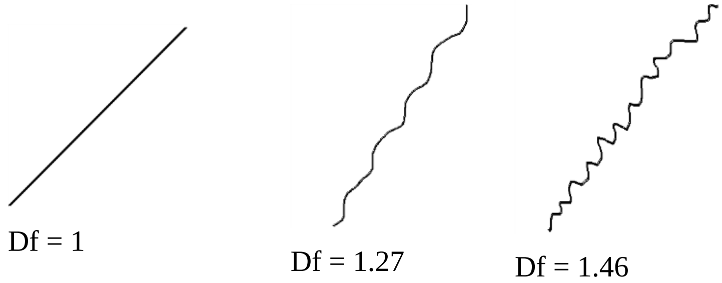


Fig. 1. The fractal dimension grows as the shape is more irregular

3. Fractal dimension and financial volatility

In the last years, there have been various attempts of using fractal analysis in financial market [3][4][14]. Fractal dimension proved to be an important instrument for two reasons: first because as Mandelbrot noticed, stock market prices are fractal objects, second, because the fractal dimension is a measure of the degree of the image's contour complexity.

For the present research, two assets were considered (using the time series prices evolution for the same period of time). Their fractal dimensions were computed using box-counting algorithm. An original software application was used: DimFr, which was developed by the author for fractal analysis purposes. The software is presented in detail in [6].

Fig. 2. The evolution of two assets prices in one year

Every time series were covered, successively, with equal squares of various sizes, then the log-log curves were traced. The covering for the first time-series is presented in Annex Fig. 1, while the corresponding log-log curve is presented below (fig. 3):

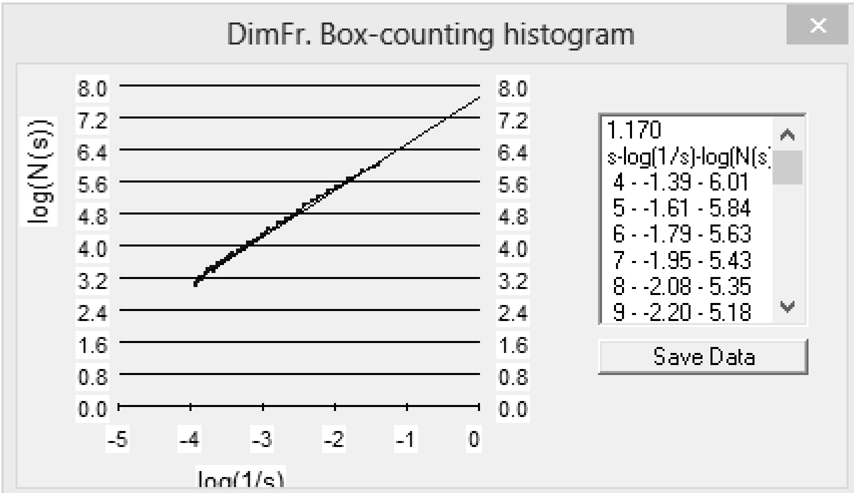


Fig. 3. The log-log curve and the s, log (1/s), log (N(s)) values for the asset (a)

Using linear regression, the slope of the log-log is computed. This yields to the value of 1.170. So, the fractal dimension for the time series above is 1.170, suggesting that the asset had a stable behaviour in that year.

The second time series has a corresponding fractal dimension of 1.324, suggesting that, even it is not a high-risk asset, its volatility is greater than the one of the first asset. The result is also suggested by the visual observation that the second asset has a more fragmented evolution than the first one.

The result is encouraging, since it suggests that the box-counting method could be used as an instrument for measuring volatility. The experiment was tested on 20 pairs of assets, and the results were confirmed by financial analysts.

Conclusions and future work

In the last decade, fractal geometry has made its way in financial analysis. The benefit use of fractals in finance was suggested even by their creator, B. Mandelbrot. Several techniques has been used, some more successful, some not.

In this paper, the box-counting algorithm was used in order to describe the volatility of an asset. It is well-known that the complexity of the form can be measured through the fractal dimension. Moreover, the time-series of a high-risk asset has a fragmented shape, and, thus, a higher complexity with a higher fractal dimension.

This yield to the idea that the fractal dimension is linked with the financial volatility: higher-risk assets having higher fractal dimensions, while stable assets having lower fractal dimensions, close to 1.

20 tests have been made on pairs of two assets; the results were confirmed by analysts: assets with lower fractal dimensions are more stable then assets with higher fractal dimension.

In a future work, the evolution of the fractal dimension will be compared with the evolution of other classical measures of volatility. For instance, the fractal dimension, given by an image processing method, will be put against a statistical measure such is standard deviation.

This paper has been financially supported by scientific research within the project entitled "PRACTICAL SCHOOL: Innovation in Higher Education and Success on the Labour Market", project identified as POSDRU/156/1.2/G/132920. The project is co-financed by the European Social Fund through the Sectorial Operational Programme for Human Resources Development 2007-2013. Investing In people!

REFERENCES

[1] http://www.investopedia.com

[2] Bucharest Stock Exchange, http://www.bvb.ro/

[3] Quant Trader, http://fractalfinance.com

[4] Fractal, SA, www.fractalerts.com

[5] Anderson, N and J Noss, 'The Fractal Market Hypothesis and its implications for the stability of financial markets', Bank of England Financial Stability Paper No. 23. (2013)

[6] Crisan D.A., Processing images using fractal techniques. Applications in medical imaging., Ed. Universitară, Bucureşti, 195 pg., ISBN (10) 973-749-130-0, (13) 978-973-749-130-5 (2006)

[7] Dobrescu, R. ; Dobrescu, M. ; Ichim, L. , Using texture and fractal analysis for classification of cell nuclei from light scattering spectroscopic images, in Hyperspectral Image and Signal Processing: Evolution in Remote Sensing (WHISPERS), 2010 2nd Workshop, pp. 1 – 4, DOI: 10.1109/WHISPERS.2010.5594874 (2010)

[8] Dobrescu, R. ; Mocanu, S., Using Fractal Dimension as Texture Discriminator for Context Base Image Retrieval, in Systems, Signals and Image Processing, 2007 and 6th EURASIP Conference focused on Speech and Image Processing, Multimedia

Communications and Services. 14th International Workshop, pp. 82 - 85DOI: 10.1109/IWSSIP.2007.4381158 (2007)

[9] Dobrescu, R., Dobrescu, M., Mocanu, St., Popescu, D. , Medical images classification for skin cancer diagnosis based on combined texture and fractal analysis, in WSEAS Transaction on Biology and Biomedicine (2010)

[10] Dobrescu, R., Ichim, L. ; Mocanu, S. ; Popescu, D., Benign and malignant breast tumors: Diagnosis using fractal measures, in Published in:System Theory, Control and Computing (ICSTCC), 2014, 18th International Conference, IEEE Explore, pp. 82-86, doi: 10.1109/ICSTCC.2014.6982395, (2014)

[11] Dobrescu, R., Ichim, L. ;Two Dimensional Modeling and Fractal Characterization of Tumor Vascular Network In Bioinformatics, Systems Biology and Intelligent Computing, 2009. IJCBS '09. International Joint Conference, pp. 483 – 486, DOI: 10.1109/IJCBS.2009.71 (2009)

[12] Dobrescu, R., Vasilescu, C., Ichim, L. – Fractal and scaling analysis in tumor growth evaluation, in WSEAS Transactions on Systems, Issue 1, vol.6, January 2007, p. 102-108 (2007)

[13] Dobrescu, R.,Crisan, D.A., Ichim, L., Diagnosis of Breast Cancer from Mammograms by Using Fractal Measures, International Journal of Medical Imaging. Vol. 1, No. 2, 2013, pp. 32-38. doi: 10.11648/j.ijmi.20130102.14 , (2013)

[14] Erik Long, The Guide to Fractal Finance, The Trade Station Guide Book, Version 3.0, USA (2002)

[15] Hasbrouck, J and Saar, G, ‘Low-latency trading’, Johnson School Research Paper Series (2011)

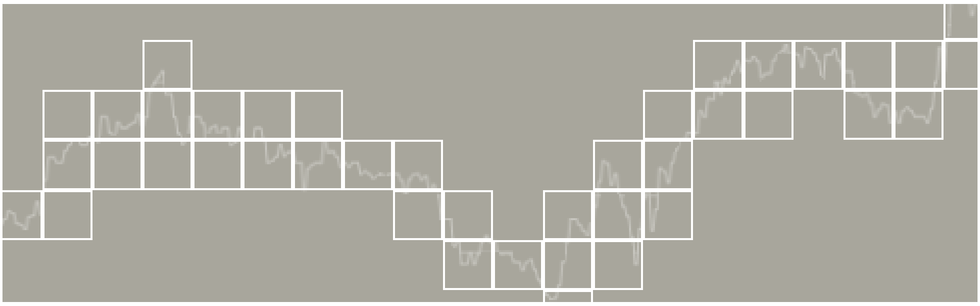
[16] Ichim, L. ; Dobrescu, R., Characterization of Tumor Angiogenesis Using Fractal Measures , in Control Systems and Computer Science (CSCS), 2013 19th International Conference, pp. 345 – 349, DOI: 10.1109/CSCS.2013.18 , (2013)

[17] Mandelbrot B. „The fractal geometry of nature”, Freeman, New York, 1983

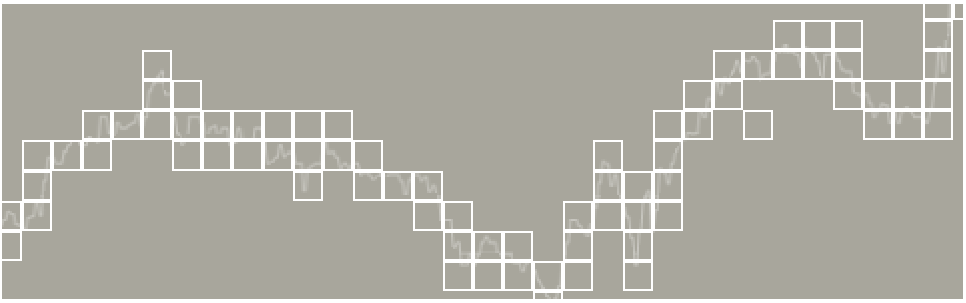
[18] Peters, E, Fractal Market Hypothesis, Wiley Finance (1991)

[19] Peters E., Fractal Market Analysis: Applying Chaos Theory to Investment and Economics (2011)

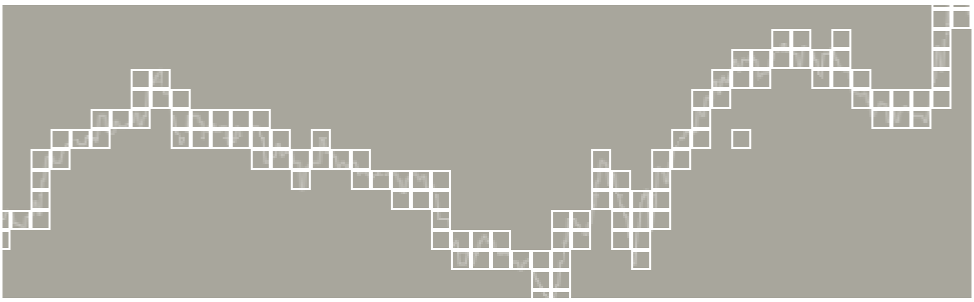
Annex. Successive coverage of the asset price evolution with equal squares



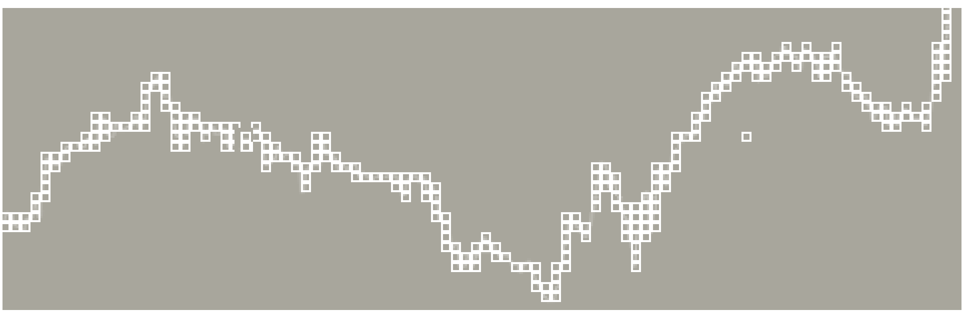
(a)



(b)



(c)



(d)

Fig. 1. Object coverage with equal squares of different side values –"s"