

**THE OUTPUT EFFECT OF STOPPING INFLATION WHEN  
VELOCITY IS TIME VARYING**

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**Abstract**

*This paper explores the role of time varying velocity on output responses to policies for reducing/stopping inflation. We study a dynamic general equilibrium model with sticky prices in which we introduce time varying velocity. Specifically, nonstationary velocity is endogenised in the model developed by Ireland (1997) for analysing optimal disinflation. The non-linear solution method reveals that, depending on velocity, the ‘disinflationary boom’ found by Ball (1994) may disappear and that early output losses may be much larger than previously thought. Indeed, we find that a gradual disinflation from a low inflation may even be undesirable given its overall negative impact on the economy*

Keywords: price stability, velocity, disinflation, output boom, optimal speed of disinflation.

**Introduction**

This paper explores the output response to a disinflationary monetary policy when velocity is time varying. The analysis takes place in an environment where the supply-side of the economy is characterized by monopolistically competitive firms and where there is rigidity in the setting of prices. The monetary policymakers are committed to price stability in the strict sense of achieving and maintaining a constant price level. This environment is familiar from recent research on monetary contractions (Ball (1994), Ireland (1997), King and Wolman (1999), and Khan, King and Wolman (2003)).

Amongst the important insights this research has provided is that, following a monetary contraction, real output initially declines below its new long run equilibrium level. Furthermore, and much more striking, is the result that a gradual disinflation may bring about a temporary output boom after the initial decline - because output may rise above its new steady state level (the so-called ‘disinflationary boom’). These output booms are not only counterintuitive but also are rarely observed in the data. Since the output effects of monetary contractions are of first order policy importance, it is not surprising that there is interest in

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exploring the robustness of these results to relaxation of key assumptions. Nicolae and Nolan (2006) relax the assumption of perfect credibility and demonstrate that the disinflationary boom may disappear in an environment characterized by imperfect credibility, depending on the speed of learning relative to the speed of disinflation. Also, Burstein (2006) allows for inflation inertia (by implementing sticky plans) and finds no disinflationary booms and, depending on the initial inflation rate, finds that early output losses may be small<sup>5</sup>.

A feature of the aforementioned new Keynesian literature is the hypothesis of constant unitary velocity essentially because money demand is not formally modelled but is postulated. Unitary velocity implies that the policymaker chooses a time path of the money supply which just supports nominal GDP while making strong assumptions about money demand behaviour. Yet, it is well known that velocity is not a constant.

As long ago as the mid 1960s, Mundell (1965) wrote that: "[t]he simplest hypothesis that velocity is constant, is clearly inadmissible when different rates of inflation are involved". More recently, the potential importance of allowing for changing velocity is being recognised in policy oriented research (see for example Orphanides and Porter (1998)) and there is ongoing research trying to construct models which can capture the variability in velocity seen in the data (see for example Hodrick et. al. (1991) and Wang and Shi (2006)). It seems that the need to appreciate and understand the implications of velocity not being constant is becoming increasingly recognised. In this paper, we specifically focus on examining the behaviour of output during disinflationary periods in a setup which allows for time varying velocity. To do this we develop a dynamic general equilibrium model with sticky prices in which we introduce time varying velocity. Given the current consensus that velocity displays nonstationary behaviour (Gould and Nelson (1974) and Friedman and Kuttner (1992), Ireland (1995)), the specific form of the relationship employed in this paper captures velocity as a nonstationary variable and nests constant velocity as a special case. We employ a non linear solution method which allows us both to explore output responses to a range of disinflationary monetary policies and to go on, by extending the solution method, to explore output responses when velocity is time varying.

The next section of this paper presents the model and the parameter values used in model calibration. Section 3 presents benchmark results familiar from the existing literature showing the output response to immediate and gradual disinflations when velocity is constant. Section 4 analyses the output responses to

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<sup>5</sup>Burstein (2006) analyses the impact of immediate disinflation only (and does not analyse gradual disinflation policies). In Ireland (1997) and Nicolae and Nolan (2006), immediate disinflation policies also yield no output boom - the booms arise only in the context of gradual disinflation. It might also be noted that whilst the models employed in Ireland (1997) and Nicolae and Nolan (2006) have both time and state dependent strategies, Burstein's model only has a state dependent strategy.

immediate and gradual disinflations when velocity is time varying. Section 5 concludes the paper.

### The model

The framework employed for this analysis extends the model developed in Ireland (1997), the component parts of which are now familiar in the literature.

The representative agent each period makes plans for consumption and leisure/labour to maximize the expected present discounted utility:

$$\sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\alpha} - 1}{1-\alpha} - \gamma N_t \right\} \quad \alpha, \gamma > 0, \quad (1)$$

which is separable in consumption and labour supply.  $\beta \in (0,1)$  is a discount factor and  $\gamma$  is the disutility of work. Consumption,  $C_t$  is defined over a continuum of goods

$$C_t = \left[ \int_0^1 c_t(i)^{\frac{b-1}{b}} di \right]^{\frac{b}{b-1}} \quad b > 0,$$

where  $c_t(i)$  is, in equilibrium, the number of units of each good  $i$  from firm  $i$  that the representative agent consumes and  $b$  is the price elasticity of demand. Labour supply,  $N_t$ , is

$$N_t = \int_0^1 n_t(i) di,$$

where  $n_t(i)$  denotes the quantity of labour supplied by the household to each firm  $i$ , at the nominal wage  $W_t$ , during each period.

Households face an aggregate price level,  $P_t$ , given by:

$$P_t = \left[ \int_0^1 p_t(i)^{1-b} di \right]^{\frac{1}{1-b}} \quad b > 0,$$

where  $p_t(i)$  is the nominal price at which firm  $i$  must sell output on demand during time  $t$ . Households supply a portion of labour to all firms which, together

with the budget constraint below (equation (2)), ensures that the marginal utility of wealth equalizes across agents.

Each period the representative household faces a budget constraint where expenditure (on non-durable consumption plus financial investment) must be less than or equal to income (financial plus labour). Each household owns an equal share of all the firms. At the beginning of each period  $t$  the household trades a number of shares,  $s_{t-1}(i)$ , at the nominal price  $Q_t(i)$ . At the end of each period  $t$  it receives the nominal dividend  $D_t(i)$  and buys new shares. Under market clearing,  $s_{t-1}(i)=1, \forall i \in [0,1]$ , in each period.

$$\begin{aligned} \int_0^1 [p_t(i)c_t(i) + Q_t(i)s_t(i)]di &\leq \\ &\leq \int_0^1 [Q_t(i)s_{t-1}(i) + D_t(i)s_t(i) + W_t(i)n_t(i)]di \end{aligned} \quad (2)$$

The household chooses  $c_t(i)$ ,  $n_t(i)$ ,  $s_t(i)$  so as to maximize (1) subject to the constraint (2) and the relevant initial and transversality conditions. Additionally, its optimal allocation across differentiated goods  $c_t(i)$  must satisfy:

$$c_t(i) = C_t \left( \frac{p_t(i)}{P_t} \right)^{-b} \quad (3)$$

In Ireland (1997), the aggregate equilibrium nominal magnitudes are determined by a quantity-theory type relation:

$$M_t V_t = \int_0^1 p_t(i)c_t(i)di = P_t C_t,$$

where  $V_t (= 1)$  is the velocity of circulation. In the model used here we relax the simplifying assumption of a constant velocity of circulation. Specifically, we introduce velocity as:

$$V_t = \Omega C_t^\delta, \quad \delta \in [0,1) \quad (4)$$

where  $\delta$  different values of the parameter  $\delta$  capture different degrees of time varying velocity and Ireland's case of a constant velocity is nested as a special case (for  $\delta = 0$ )<sup>6</sup>. For any value of  $\delta \in (0,1)$  velocity is time varying. Equation (4)

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<sup>6</sup> For simplicity  $\Omega$  is here set equal to unity.

describes the consumption velocity of money. This reflects empirical evidence from the money demand literature that aggregate consumption is the preferred proxy for the scale variable (Mankiw and Summers (1986)) and is consistent with the focus of the more recent search model approach to the velocity of money (Wang and Shi (2006)). We also draw on evidence that consumption, like velocity, displays nonstationary behaviour (Mehra (1988a), Mehra and Prescott (1984, 1985, 1988)) and the specific functional form adopted here has empirical as well as theoretical support (Basu and Dua (1996) and Basu and Salyer (2001))<sup>7</sup>.

Importantly, velocity is now nonstationary and endogenous to the model. The quantity theory relation can now be written:

$$M_t = P_t C_t^{1-\delta}. \quad (5)$$

The agent solves the maximization problem yielding the following first order conditions:

$$C_t^{-\alpha} = \lambda_t P_t; \quad (6)$$

$$\gamma = \lambda_t W_t; \quad (7)$$

(from (6) and (7))

$$W_t = \gamma P_t C_t^\alpha. \quad (8)$$

And for all  $i$

$$Q_t(i) = D_t(i) + \beta(\lambda_{t+1}/\lambda_t) Q_{t+1}(i), \quad (9)$$

where  $\lambda_t$  is an unknown multiplier associated with the budget constraint (2).

For the corporate sector, the supply-side of the economy consists of monopolistically competitive firms and there is price rigidity. A continuum of firms indexed by  $i$  over the unit interval, each produces a different, perishable consumption good, indexed by  $i \in [0,1]$ , where firm  $i$  produces good  $i$ . Each firm  $i$  sells shares, at the beginning of each period  $t$ , at the nominal price  $Q_t(i)$ , and pays, at the end of the period, the nominal dividend  $D_t(i)$ .

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<sup>7</sup> A full explanation of the microfoundations of this velocity function is an interesting exercise in its own right but is beyond the scope of the current paper. The approach taken here is consistent with the usual assumption that velocity shocks are measured as i.i.d. shocks to an AR(1) process.  $C_t$  is autocorrelated in this model, therefore  $V_t = \rho V_{t-1} + \varepsilon_t$ .

We assume a simple linear production technology  $y_t(i)=l_t(i)$ , where  $y_t(i)$  and  $l_t(i)$  are the output of firm  $i$  and the labour used to produce it, respectively.  $Y_t$  is aggregate output. Equilibrium returns to shareholders at time  $t$  for firm  $i$  are given by:

$$D_t(i) = [p_t(i) - W_t(i)] \left( \frac{p_t(i)}{M_t} \right)^{-b} C_t^{1-b(1-\delta)} - I_t(i)W_t(i)k, \quad (10)$$

where

$$I_t(i) = \begin{cases} 1, & \text{if the firm pays the cost of price} \\ & \text{adjustment } k; \\ 0, & \text{if the firm does not pay the cost } k. \end{cases}$$

Costly price adjustment is central to this model in which time-dependant and state-dependant strategies are both present. Firms are divided into two categories, such that at time  $t$ , firms from the first category can freely change their prices,  $p_{1,t}(i)$ , while firms belonging to the second category must sell output at the same price set a period before,  $p_{2,t}(i) = p_{2,t-1}(i)$ , unless they pay the fixed cost  $k > 0$ , measured in terms of labour. At time  $t + 1$ , the roles are reversed and the first category of firms keeps prices unchanged,  $p_{1,t+1}(i) = p_{1,t}(i)$  unless they are willing to pay the fixed cost  $k$ , while the second category of firms can freely set new prices.

Firms are constantly re-evaluating their pricing strategy, weighing the benefits of holding prices fixed versus the alternative of changing prices and incurring the fixed penalty. At moment  $t$  the firms that can freely change price are able to choose between two strategies, depending on whether the inflation rate is moderate or high. At moderate rates of inflation, they are more likely to keep their prices constant for two periods and hence avoid the cost  $k$  (single price strategy). On the other hand, in the case of a high inflation, or in the face of sharp changes in the monetary stance, firms are more likely to choose a new price and pay the cost  $k$  (two price strategy). The price-setting decision at time  $t$  maximises the return to shareholders.

The equilibrium in the model is given by the market clearance conditions for the three markets present in this model (goods market, labour market and asset market). Clearance in two markets assures clearance in the third. From the market clearance conditions for the goods and labour markets we have:

$$C_t = Y_t = L_t. \quad (11)$$

The clearance condition for the asset market is  $s_{t-1}(i) = 1, \forall i \in [0,1]$ , in each period.

Under the single price strategy, firm  $i$  chooses the price  $p_t(i)$  to maximize the expression:

$$\Pi_t(i) = D_t(i) + \beta(\lambda_{t+1}/\lambda_t) D_{t+1}(i), \quad (12)$$

which follows from (9) and implies that prices are set to maximize market value. Substituting (5) and (8) into (10), and then this into equation (12), yields the price firm  $i$  will use for two consecutive time periods:

$$p_t(i) = \frac{b}{b-1} \gamma \frac{M_t^b Y_t^{1-b(1-\delta)} + \beta M_{t+1}^b Y_{t+1}^{1-b(1-\delta)}}{M_t^{b-1} Y_t^{2-b(1-\delta)-\alpha-\delta} + \beta M_{t+1}^{b-1} Y_{t+1}^{2-b(1-\delta)-\alpha-\delta}} \quad (13)$$

This equation, familiar from the New Keynesian economics literature, shows that the optimal price is a function of current and future anticipated demand and cost conditions; and that, in steady state, price is a fixed mark-up over marginal costs. As is familiar in models of monopolistic competition, the markup is constant and determined by the elasticity of demand (that is, it is tied down via the preference side of the model): the lower the elasticity, the higher the mark-up.

Under the two price strategy, firm  $i$  chooses the price  $p_t(i)$  to maximise the expression:

$$\Pi_t(i) = D_t(i) \quad (14)$$

and now the optimising price is:

$$p_t(i) = \frac{b}{b-1} \gamma \frac{M_t}{Y_t^{1-\alpha-\delta}} \quad (15)$$

Again, prices are a mark-up, but now only current period demand and cost conditions are relevant since only current dividend matters.

## Monetary Policy

The disinflationary policy employed in this paper follows the approach adopted by Ball (1994), Ireland (1997) and Nicolae and Nolan (2006). The monetary policy is designed to bring money growth to zero over some time horizon. Specifically, at period 0, the authorities make a surprise announcement

about the path for the money supply,  $\{M_t\}_{t=0}^T$ , such that by time period  $T$  inflation will be zero. This announced path for the money supply, implies a decrease in the growth rate of the money supply.

Let

$$\theta_t = \frac{M_t}{M_{t-1}}$$

denote the gross rate at which the money supply increases at time  $t$ . We adopt a disinflationary process of the following sort:

$$\theta_t = \theta_{t-1} - \varphi^{T-1}(\pi_t - \pi^*), \quad \varphi \in (0,1),$$

where  $\pi_t$  is the initial rate of inflation from which the disinflation process starts,  $\pi^*$  is the final (target) inflation to be set here at  $\pi^* = 1$  and  $\theta_t > 1$ , for any value of  $t$  from 0 to  $T - 1$ .

An horizon of time  $T = 1$  entails immediate disinflation, while for  $T > 1$  the policymakers engineer a more gradual path towards price stability. To facilitate comparison with the existing literature we employ a linear disinflationary policy following Ireland (1997) and Nicolae and Nolan (2006) which we obtain for

$$\varphi = \left(\frac{1}{T}\right)^{\frac{1}{T-1}}.$$

### Model Calibration

This section presents the calibration of the model. To facilitate comparison with the existing literature, we employ parameter values drawn from the wider literature, as used in Nicolae and Nolan (2006). For ease of reference, Table 1 sets out the parameter values used in the calibration. We allow the newly introduced parameter  $\delta$  to take a number of different values in order to explore the effect of time varying velocity on output (Ireland's case ( $\delta = 0$ ) is a special case of the work carried out here).

Parameter	Value	Description
$\alpha$	0.1	intertemporal elasticity of substitution; (value as in Ball, Mankiw and Romer, 1988)
$b$	6	price elasticity of demand; (value as in Rotemberg and Woodford, 1992)
$k$	0.1075	cost of price adjustment; (value as in Ireland, 1997)



$\beta$	0.97	discount factor; each interval of time corresponds to 6 months; (value as in Ball and Mankiw, 1994)
$\gamma$	1	degree of disutility from work; (value as in Nicolae and Nolan, 2006)
$\delta$	[0,1)	degree of time varying velocity;

Table 1. Parameter values used in the model calibration.

In the following section, we present benchmark results from the existing literature. These describe the behaviour of output during immediate and gradual disinflations starting from both low and high initial inflation rates, where velocity is assumed constant. The subsequent section presents the behaviour of output for all of these same cases but when velocity is assumed to be time varying.

### Benchmark results

This section presents results familiar from the literature for the specific case where velocity is assumed constant.

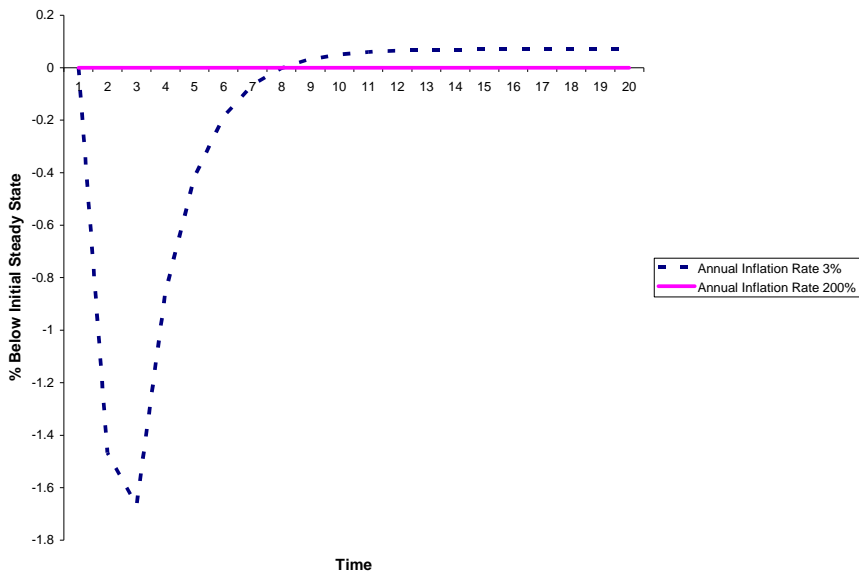


Figure 1. Benchmark Result (Ireland, 1997): Output effect of immediate disinflation of a 'small' (3%) and a 'big' (200%) initial annual inflation rate.

Figure 1. shows two key results: i) that immediate ( $T = 1$ ) disinflation from a low (3%) inflation rate brings about a significant early output loss (some 1.47% in the first period and 1.67% in the second period) before reaching its new steady

state level; and ii) that immediate disinflation from a high (200%) inflation has no output effect.

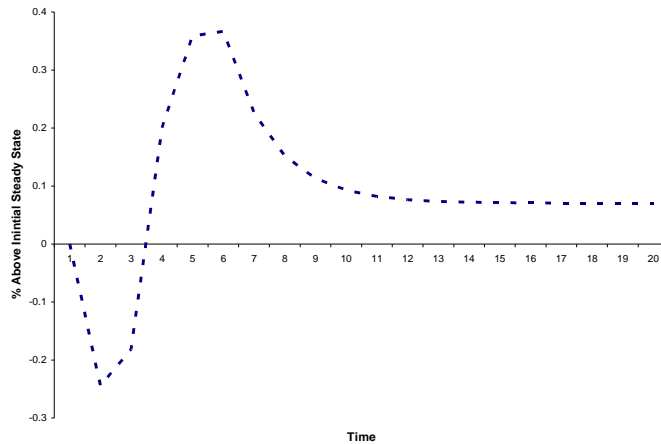


Figure 2. Benchmark Result (Ireland, 1997): Output effect of a gradual disinflation from a 'small' (3%) initial annual inflation rate.

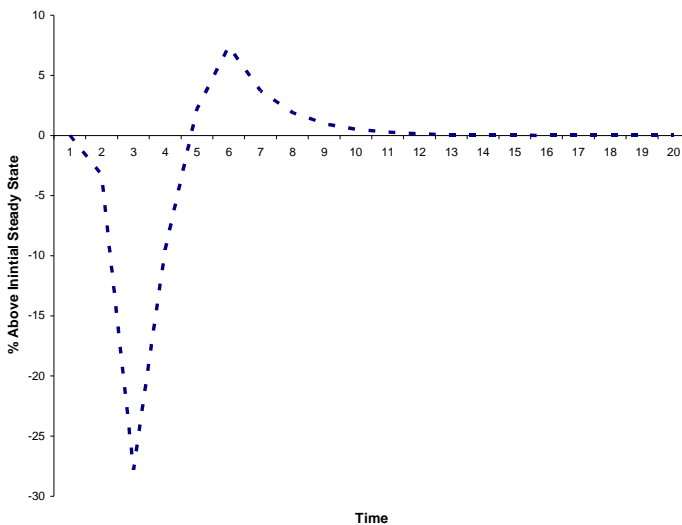


Figure 3. Benchmark Result (Ireland, 1997): Output effect of a gradual disinflation from a 'big' (200%) initial annual inflation rate.

Figure 2 sets out the case where disinflation is gradual ( $T = 6$ ) and focuses on disinflating from a low (3%) initial inflation rate. There are two important features to note: i) the early output loss is less than that under the immediate disinflation

(now 0.2% in the first period); and ii) after the early fall in output, there is a substantive (compensatory) output boom before a new steady state is reached<sup>8</sup>.

Figure 3 presents the output effect of disinflating gradually ( $T = 6$ ) from a high (200%) initial inflation rate. There is now a substantive early output loss (27% below the initial steady state); and again an output boom, but only part compensatory, before reaching the new steady state.

These benchmark images underlie the now well known policy conclusion that high inflations are best ended abruptly and low inflations are best ended gradually. The key issue is the impact on the real economy. Three elements are important here: (1) the extent of output losses in the early periods after a monetary contraction; (2) the existence (or otherwise) of a temporary output boom (defined as output rising above the new steady state); and (3) whether early output losses are compensated over some reasonable time horizon.

This paper explores these issues when the model assumption of constant velocity is relaxed. In order to do this, the nonlinear solution method is extended to incorporate time varying velocity. We will see that introducing time varying velocity to the modelling framework prompts us to modify our stance on some of these issues.

### Output effects of immediate and gradual disinflation with time varying velocity

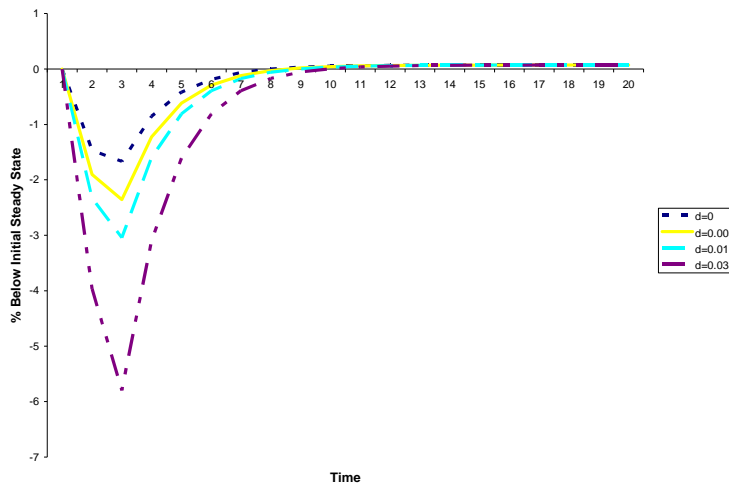


Figure 4. Time Varying Velocity Result: Output effect of immediate disinflation from a low initial annual inflation rate (3%).

<sup>8</sup> Such disinflationary booms are typically understood as follows. Under perfect credibility, agents respond in advance of the change in policy by lowering their prices, knowing that, inflation is going to be lower in the future. Because agents set prices for two periods, and because inflation will be lower in the future, they set lower prices today, inducing a boom (Ball (1994)).

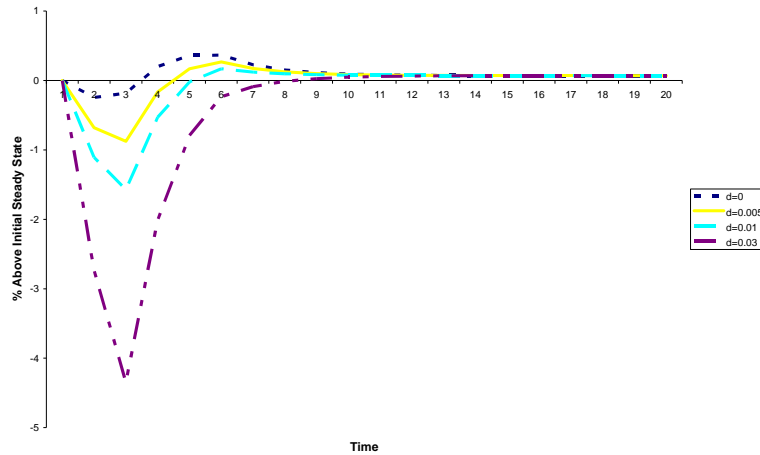


Figure 5. Time Varying Velocity Result: Output effect of gradual disinflation from a low initial annual inflation rate (3%).

Figure 4 sets out the output effect of an immediate disinflation ( $T = 1$ ) from a low (3%) initial annual inflation rate. Different values for  $\delta$  capture different degrees of time varying velocity ( $\delta = 0$  reflects the benchmark case set out in the previous section and the (dashed) output path corresponds to that seen in Figure 1. Higher values of  $\delta$  reflect higher degrees of time varying velocity. It can be seen that the effect of introducing varying velocity is to increase the early output loss. To see why this comes about, we refer to the price setting strategies set out in equations (13) and (15). The time varying velocity parameter  $\delta$  enters the price setting strategies for both types of firms augmenting the overall output effect. This process is discussed in more detail, after considering the output response to a gradual ( $T = 6$ ) disinflation from a low initial 3% inflation rate.

In Figure 5, again, the dashed line reflects the benchmark case when velocity is constant ( $\delta = 0$ ), as seen in Figure 2. As in the previous case of immediate disinflation, we see that introducing time varying velocity to the model has induced greater output losses: the higher the value of  $\delta$ , the lower the output falls below its initial steady-state level in the early period. However, in this case, velocity seems to have one additional effect. In the benchmark case of gradual disinflation with constant velocity, we saw that, after the initial fall, output not only picked up but also rose above its new steady state level, staying above for some time before returning to its new steady-state equilibrium (the output boom). However, for velocity variability characterized by  $\delta^* \in (0.01, 0.02)$  we see that, after the initial fall, output recovers but never rises above the new steady state level. Moreover, this is so for all yet higher values of  $\delta$ . For any  $\delta > \delta^*$ , output fails to reach any level above the new steady-state. Although output reaches its new steady-state at about the same time (4-5 years) regardless of the velocity parameter value ( $\delta$ ), the higher is velocity the greater is the output loss and the

greater is the possibility that there is no output boom. This raises a key question about whether gradual disinflation is beneficial. With greater output losses for some values of  $\delta$ , there is the possibility that they might not be compensated through a disinflationary boom.

To explore this issue further, we construct a crude measure of the overall impact on output by projecting forward over a 30 year time horizon and calculating the net output gain. Table 2 sets out the value of the area between the 'output path' and the x axis for a range of  $\delta$  values. The area below the axis gives the output loss, and above the axis gives the output gain. The absolute size of the overall impact is noted in the final column and defined to be the net output gain. We can see that for sufficiently high values of  $\delta$  the overall impact on output is negative. (If we were to calculate present values, overall net losses would arise at even lower levels of  $\delta$ ).

$\Delta$	Loss	Gain	Net Output Gain
0	0.42	4.97	4.55
0.001	0.65	4.82	4.17
0.005	1.72	4.38	2.66
0.01	3.22	4.00	0.77
0.02	6.60	3.61	2.99
0.03	10.22	3.49	6.73
0.05	17.56	3.40	14.15

Table 2. Overall impact on real output of a gradual disinflation from a 3% initial annual inflation rate for different values of the velocity parameter ( $\delta$ ).

In the light of these results, Ireland's (1997) conclusion that small inflations are best ended gradually may need to be qualified: it seems that even disinflating a low inflation gradually may be undesirable since the net 'overall impact' on the real economy may be negative. This shift in potential policy conclusion is solely attributable to the introduction of time varying velocity so it is helpful to discuss its role in the (behavioural) context of the model. After the disinflation is announced at  $t = 0$ , at  $t = 1$  the firms that changed price last period now keep their price fixed, but the other set of firms respond by adjusting their prices. When they solve their optimization problem to maximize their profits, firms take the nominal money supply  $M_t$ , the aggregate general level of prices  $P_t$  and  $V_t = \Omega C_t^\delta$

as given. In equilibrium, we know that  $\left(\frac{M_t}{P_t}\right)^{\frac{1}{1-\delta}}$  has to be consistent with the individual firm choice. Thus, each price  $p_t(i)$ , must be optimal such that  $C_t$  must

equal  $\left(\frac{M_t}{P_t}\right)^{\frac{1}{1-\delta}}$  (see equation (5)). For  $\delta > 0$  real money balances ration household demand, prices must rise for firms to maximise profits. From a simple manipulation of (5), we can get some feel for the role of  $\delta$  and how this affects consumption's response to the disinflation. Taking logs one gets:

$$\ln C_t = \frac{1}{1-\delta} (\ln M_t - \ln P_t).$$

Partially differentiating with respect to  $M_t$ , yields

$$\frac{d \ln C_t}{d \ln M_t} = \frac{1}{1-\delta} > 0,$$

which shows that when  $\delta > 0$ , a change in  $M_t$  induces an even greater change in  $C_t$  than when  $\delta = 0$ . Giving the equilibrium condition (11), this explains the extra real cost imposed by time varying velocity following a monetary contraction. This also explains the higher fall in output following immediately after the announcement of disinflation when time varying velocity is present. Following the announcement of the change in policy, the economy moves from the initial steady state to the disinflationary policy path whereby the announced decrease in  $M_t$  induces a proportionally higher decrease in output.

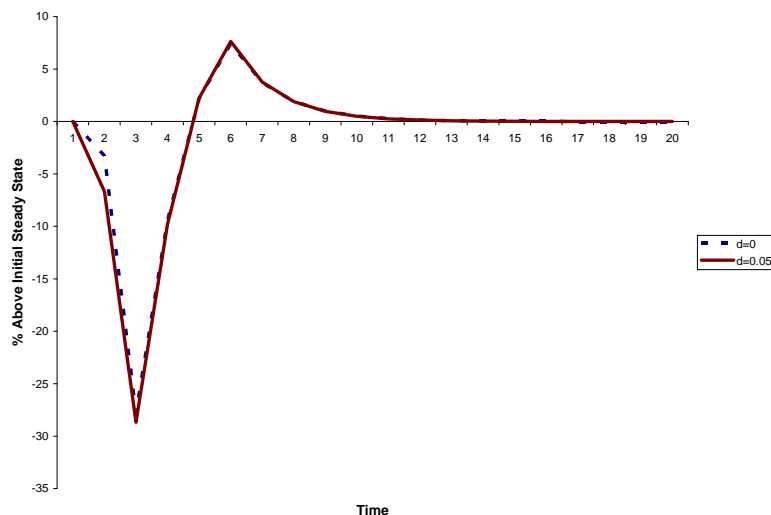


Figure 6. Time Varying Velocity Result: Output effect of immediate and gradual disinflation from a high initial annual inflation rate (200%).

We now turn to consider the case where disinflation is from a high (200%) initial inflation rate. Figure 6 sets out the output path resulting from each of an immediate disinflation and a gradual disinflation. There is no impact of time varying velocity in the case of an immediate disinflation ( $\delta = 0$  and  $\delta = 0.05$  shown). At very high inflation rates, both sets of firms are following the two price strategy because the costs of adjustment are outweighed by the benefits. Not only is inflation ended abruptly but also, adjustment is so fast that there is no scope for velocity to have an impact.

More interesting is the case of gradual disinflation. In Figure 6, the output path with time varying velocity ( $\delta = 0.05$ ) looks very similar to the benchmark case ( $\delta = 0$ ). However, in the first period, the output loss is more marked. The reason for this is akin to the output effect we have seen when disinflation was carried out gradually from a low initial inflation rate. We have seen that when disinflation is gradual,  $\delta$  has a role to play and its role is to reduce output more. This result seems to reinforce Ireland's conclusion that gradual disinflation from a high initial rate is not to be recommended. We therefore turn our attention to consider gradual disinflation from a range of lower inflation rates in more detail. Specifically, we seek to establish the impact of time varying velocity on the optimal speed of disinflation from a range of initial inflation rates.

## **Discussion and conclusions**

Perhaps the most dramatic finding from recent research on monetary contractions is that a gradual disinflation may bring about a 'disinflationary output boom'. These disinflationary output booms were first recorded in the much cited paper by Ball (1994); and more recent literature (in which firms are monopolistically competitive and there is rigidity in prices) consistently finds such booms (see for example, Ireland (1997), King and Wolman (1999), Khan, King and Wolman (2003)). Ball (1994) attributes the disinflationary boom to the assumption of perfect credibility. Nicolae and Nolan (2006) relax the assumption of perfect credibility and find that, whilst imperfect credibility may make these booms disappear, it is not a sufficient condition: their (dis)appearance depends on the speed of learning relative to the speed of disinflation. In this paper, we relax another assumption common in this literature, that of constant velocity. We find that even with perfect foresight the disinflationary booms may disappear, but now this is a result of time varying velocity. We find that output boom (dis)appearance depends on velocity.

This is not the only effect of relaxing the constant velocity assumption. Firstly, we find that the early output loss that follows a disinflationary policy announcement is considerably larger when time varying velocity is introduced to the model; and this output loss may not be compensated by later output gains. As

a result, we find that we cannot unconditionally endorse Ireland's policy recommendation that small inflations are best disinflated gradually. We find that a gradual disinflation from a small inflation may result in an overall output loss, bringing into question the desirability of any disinflationary policy action in some cases. It seems that some of the familiar results and policy implications from influential work on stopping inflations are not robust to some modifications of the modelling framework. Given the practical importance of the underlying policy issue, further research on model specification would seem warranted.

## References

Ball, Laurence, N. Gregory Mankiw and David Romer (1988), "The New Keynesian

Economics and the Output-Inflation Trade-Off", **Brookings Papers on Economic Activity**, Vol. 1, pp. 1-65.

Ball, Laurence (1994), "Credible Disinflation with Staggered Price-Settings", **The American**

**Economic Review**, Vol. 84, No. 1, pp. 282-289.

Ball, Laurence and N. Gregory Mankiw (1994), "Asymmetric Price Adjustment and

Economic Fluctuations", **Economic Journal**, Vol. 104, March, pp. 247-261.

Basu, Parantap, and Pami Dua (1996), "The Behaviour of Velocity and Nominal Interest

Rates in a Cash-in-Advance Model", **Journal of Macroeconomics**, 18(3), pp. 463--478.

Basu, Parantap and Kevin D. Salyer (2001), "Modeling Money Demand in Growing

Economies", **Bulletin of Economic Research**, Vol. 53(1), pp. 53-60.

Burstein, Ariel (2006), "Inflation and Output Dynamics with State-dependent Pricing

Decisions", **Journal of Monetary Economics**, Vol. 53, pp. 1235-1257.

Friedman, Benjamin M., and Kenneth N. Kuttner (1992), "Money, Income, Prices and

Interest Rates", **The American Economic Review**, Vol. 82, pp 472-92

Gould, John P. and Charles R. Nelson (1974), "The Stochastic Structure of the Velocity of

Money", **The American Economic Review**, Vol. 64, No. 3. pp. 405-418.

Hodrick, Robert, Narayana Kocherlakota, Deborah Lucas (1991), The Variability of Velocity

in Cash in Advance Models, **The Journal of Political Economy**, Vol 99. No.2 pp. 358-384.



- Ireland, Peter N. (1995), "Financial Innovation and the Demand for Money", **Journal of Money, Credit and Banking**, Vol. 27, No. 1 (Feb., 1995), pp. 107-123.
- Ireland, Peter N. (1997), "Stopping Inflation, Big and Small", **Journal of Money, Credit and Banking**, Vol. 29, No.4 (November, Part 2), pp. 759-775.
- Khan, Aubik, Robert G. King and Alexander L. Wolman (2003), "Optimal Monetary Policy", **Review of Economic Studies**, Vol. 70, Issue 4, pp. 825-860.
- King, Robert G. and Alexander L. Wolman (1999), "What Should the Monetary Authority Do When Prices are Sticky?" in **Monetary Policy Rules**, edited by John Taylor, University of Chicago Press for NBER, Chicago pp. 349-404.
- Mankiw, N. Gregory and Lawrence Summers (1986), "Money Demand and the Effects of Fiscal Policy", **Journal of Money, Credit and Banking**, Vol. 18, No. 44, pp. 415-429.
- Mehra, Rajnish (1988), "On the Existence and Representation of Equilibrium in an Economy with Growth and Nonstationary Consumption", **International Economics Review**, Vol 29, No.1, February, pp. 131-135.
- Mehra, Rajnish and Edward C. Prescott (1984), "Asset Prices with Nonstationary Consumption", **Working Paper, Graduate School of Business**, NY: Columbia University.
- Mehra, Rajnish and Edward C. Prescott (1985), "The Equity Premium: A Puzzle," **Journal of Monetary Economics**, Vol. 15, pp. 145-161.
- Mehra, Rajnish and Edward C. Prescott (1988), "The Equity Risk Premium: A Solution?", **Journal of Monetary Economics**, Vol. 22, pp. 133-136.
- Mundell, Robert A. (1965), "Growth, Stability and Inflationary Finance", **Journal of Political Economy**, Vol 73, No. 2 , pp. 97-109.
- Nicolae, Anamaria and Charles Nolan (2006), "The Impact of Imperfect Credibility in a Transition to Price Stability", **Journal of Money, Credit and Banking**, Vol. 38, No. 1, pp. 47-66.
- Orphanides, Athanasios and Richard Porter (1998), P\* Revisited: Money-Based Inflation Forecasts with a Changing Equilibrium Velocity, **Finance and Economics Discussion Paper Series**, 1198-26, Washington, DC: Federal Reserves Board.

Rotemberg, Julio J. and Michael Woodford (1992), "Oligopolistic Pricing and the Effects of

Aggregate Demand on Economic Activity", **Journal of Political Economy**, Vol. 100, December, pp. 1153-1207.

Wang, Weimin and Shouyong Shi (2006), The Variability of Velocity of Money in a Search

Model, **Journal of Monetary Economics**, Vol. 53, pp 547-471.